Supplement to “Stability of Causal Inference”

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1 Proof of Proposition 1.1

In this section, we provide a proof of Proposition 1.1 that was omitted from the main body of the paper. We follow the same notation as in the paper. However, the statement of the proposition is reproduced here for quick reference.

Proposition 1.1. *Consider a semi-Markovian graph $G = (V, E, U, H)$ and a distribution $P(V, U)$ respecting it. Let $R = \{e_i = (A_i, V_i) \mid 1 \le i \le q\}$ be a set of k edges in $E \cup H$ such that e_i is ϵ_i-weak, with $\epsilon := \sum_{i=1}^{k} \epsilon_i$. Suppose that X, Y are disjoint subsets of V for which $\mathbb{P}(Y \mid \mathrm{do}\, X)$ is not identifiable in G, but identifiable in $G' = (V, E \setminus R, U, H \setminus R)$. Then there exists a distribution $\tilde{P}(V, U)$ respecting G' such that*

$$-\epsilon \le \log \frac{\tilde{P}(V)}{P(V)} \le \epsilon, \quad \textit{and} \quad -\epsilon \le \log \frac{\tilde{P}(Y \mid \mathrm{do}\, X)}{P(Y \mid \mathrm{do}\, X)} \le \epsilon.$$

Note that $\tilde{P}(Y \mid \mathrm{do}\, X)$ is computable (by the algorithms of Shpitser and Pearl (2006) and Huang and Valtorta (2008)) given $\tilde{P}(V)$, but $P(Y \mid \mathrm{do}\, X)$ is not even uniquely determined given only the observed marginal $P(V)$.

Proof of Proposition 1.1. We define $\tilde{P}$ on (V, U) by giving an explicit factorization which respects G' by construction. We first define $\tilde{P}(U) = P(U)$, so that P and $\tilde{P}$ agree when restricted to the hidden variables. The weakness of the edges removed from G to obtain G' only plays a part in defining the factorization on the visible nodes in V. Let B be a vertex in V, and let $A_1, A_2, \ldots, A_k$ be the (possibly empty) set of vertices in $U \cup V$ such that among the k edges removed from G to obtain G', those incident on B are $(A_1, B), (A_2, B), \ldots, (A_l, B)$. Let $\epsilon_{B_1}, \epsilon_{B_2}, \ldots, \epsilon_{B_l}$ be such that the edge (A_i, B) is ϵ_{B_i}-weak. Let $\Xi(B)$ denote the set of parents of B in $U \cup V$ disjoint from $\{A_1, A_2, \ldots, A_l\}$. We then define:

$$\tilde{P}(B = b \mid \Xi(B) = \xi) = P(B = b \mid \Xi(B) = \xi, A_i = a_i, 1 \le i \le l), \tag{1}$$

where b, ξ are values in the domain of B and $\Xi(B)$ respectively, and a_i are *arbitrary* values in the domain of the A_i. From the definition of weakness, we then have, for all b, ξ in the domain of B and $\Xi(B)$ respectively, and for all a_i' in the domain of the A_i:

$$\left|\log \tilde{P}(B = b \mid \Xi(B) = \xi) - \log P(B = b \mid \Xi(B) = \xi, A_i = a_i', 1 \le i \le l)\right| \le \sum_{i=1}^{l} \epsilon_{B_i}. \tag{2}$$

We now define $\tilde{P}(V, U)$ in the standard way by factorizing in terms of the conditional probabilities defined in eq. (1): it then respects G' by construction. Using eq. (2), we have, for every u and v in the domain of U and V respectively,

$$\left|\log \tilde{P}(V = v, U = u) - \log P(V = v, U = u)\right| \le \sum_{i=1}^{1} \epsilon_i = \epsilon. \tag{3}$$

The claim of the proposition comparing $P(V)$ and $\tilde{P}(V)$ now follows from the fact that $P(V = v)$ and $\tilde{P}(V = v)$ are both obtained by marginalizing P and $\tilde{P}$ over the domain of U, and the latter are ϵ-close by eq. (3). The proof of the claim comparing $P(X \mid \mathrm{do}(Y))$ and $\tilde{P}(X \mid \mathrm{do}\, Y)$ follows in exactly the same fashion starting from eq. (1), by using the factorizations of these quantities in terms of the conditional probabilities appearing in eq. (1). □

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References

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