
PAC-Bayesian Inequalities for Martingales

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Abstract

We present a set of high-probability inequalities that control the concentration of weighted averages of multiple (possibly uncountably many) simultaneously evolving and interdependent martingales. Our results extend the PAC-Bayesian analysis in learning theory from the i.i.d. setting to martingales opening the way for its application in reinforcement learning and other interactive learning domains, as well as many other domains in probability theory and statistics, where martingales are encountered.

We also present a comparison inequality that bounds the expectation of a convex function of a martingale difference sequence shifted to the $[0, 1]$ interval by the expectation of the same function of independent Bernoulli variables. This inequality is applied to derive a tighter analog of Hoeffding-Azuma's inequality.

For the complete paper see Seldin et al. (2012).

References

Yevgeny Seldin, François Laviolette, Nicolò Cesa-Bianchi, John Shawe-Taylor, and Peter Auer. PAC-Bayesian inequalities for martingales. *IEEE Transactions on Information Theory*, 2012. Accepted. Preprint available at <http://arxiv.org/abs/1110.6886>.

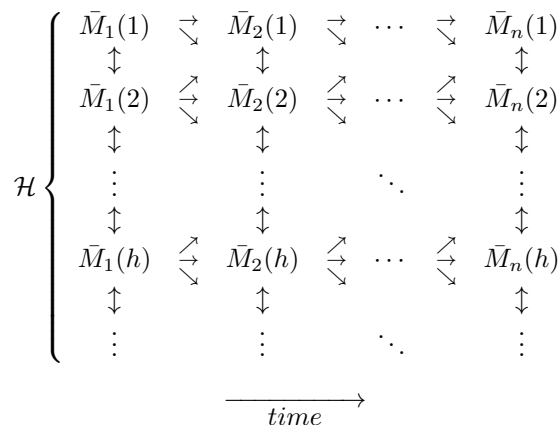


Figure 1: Illustration of an infinite set of simultaneously evolving and interdependent martingales. For a fixed h , the sequence $\bar{M}_1(h), \bar{M}_2(h), \dots, \bar{M}_n(h)$ is a martingale. \mathcal{H} is a set (possibly uncountably infinite) that indexes the individual martingales. The arrows represent the dependencies between the values of the martingales: the value of the h -th martingale at time i , denoted by $\bar{M}_i(h)$, depends on $\bar{M}_j(h')$ for all $j \leq i$ and $h' \in \mathcal{H}$ (everything that is “before” and “concurrent” with $\bar{M}_i(h)$ in time; some of the arrows are omitted for clarity). A mean value of the martingales with respect to a distribution ρ over \mathcal{H} (or a probability density function, if \mathcal{H} is uncountably infinite) is given by $\langle \bar{M}_n, \rho \rangle$. Our high-probability inequalities bound $|\langle \bar{M}_n, \rho \rangle|$ simultaneously for a large class of ρ .