Introduction to Parallel and High-Performance Computing

(with Machine-Learning applications)

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Part 1

Parallel computing basics and parallel algorithm analysis

Part 2

Parallel algorithms for Building a Classifier

Part 1: Parallel and High-Performance Computing

- Why parallel computing?

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- Parallel computing platforms

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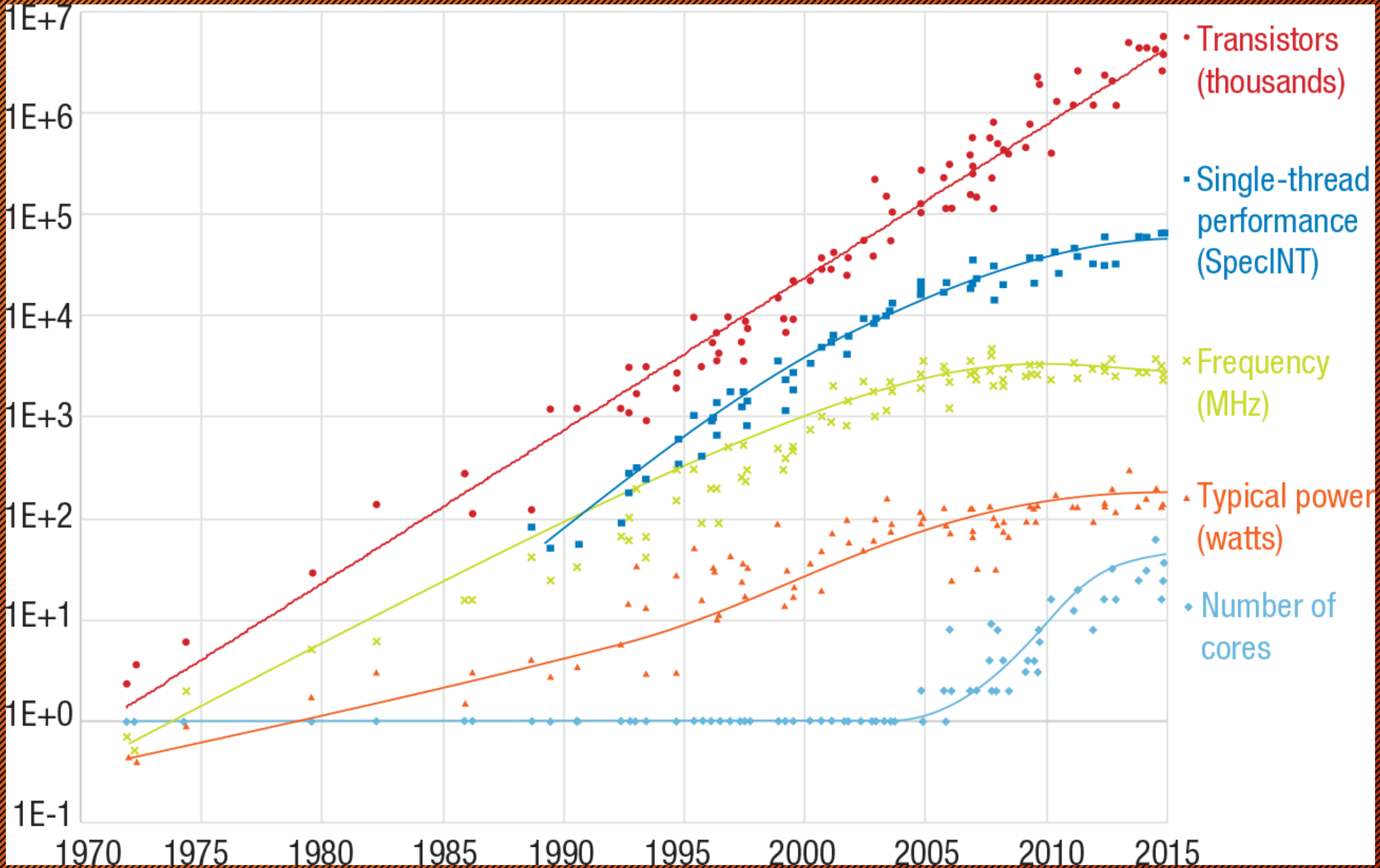
- Parallel programming models
- Parallel algorithm analysis

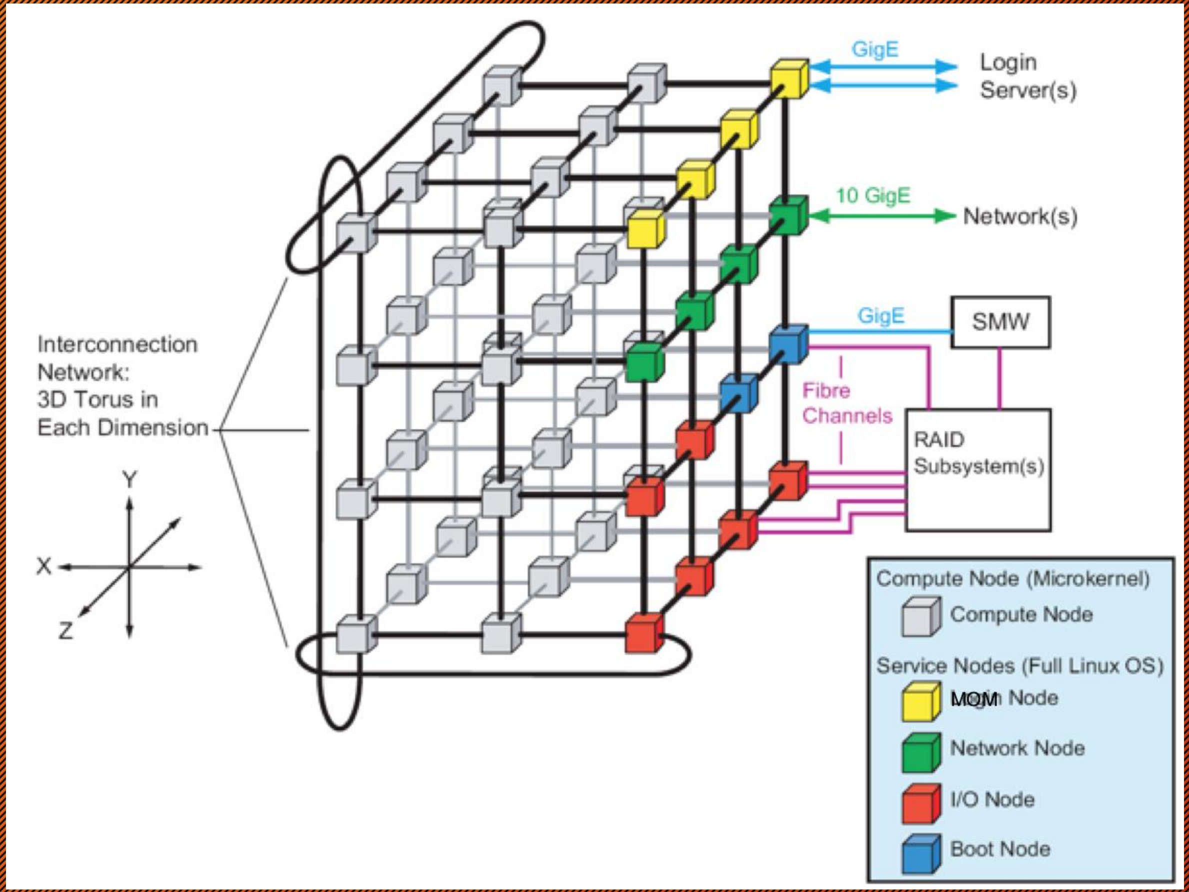
Why parallel computing?



Microprocessor trends: 1972--2015

Kirk M. Bresniker, Sharad Singhal, R. Stanley Williams, "Adapting to Thrive in a New Economy of Memory Abundance",*Computer*, vol.48, no. 12, pp. 44-53, Dec. 2015, doi:10.1109/MC.2015.368

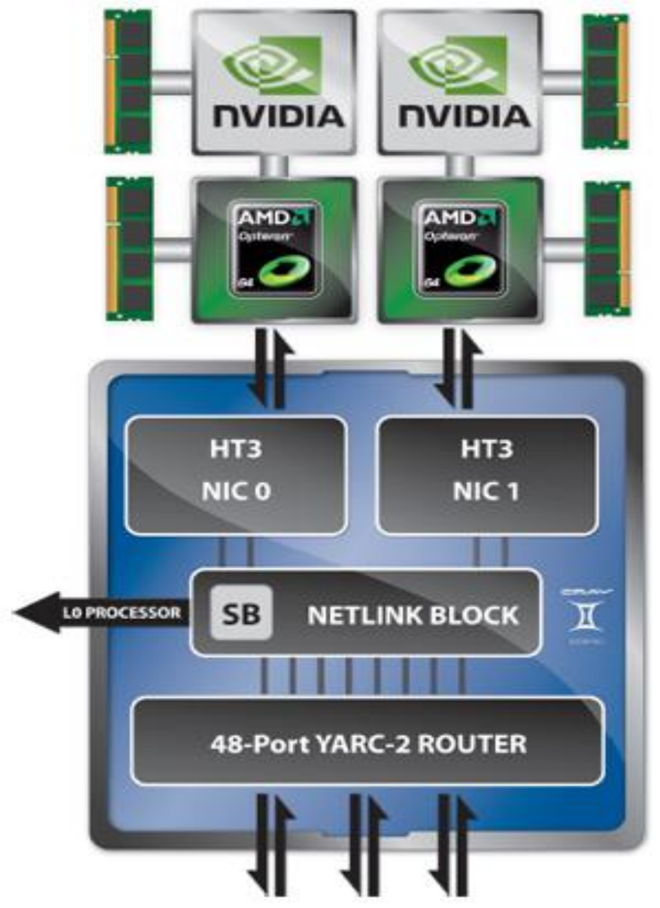
Parallel computing platforms



Highest level of parallelism:

- Compute nodes on an interconnection network
- Possibly, thousands of nodes
- Distributed memory
- Distributed or shared address space
- Scalability analysis crucial

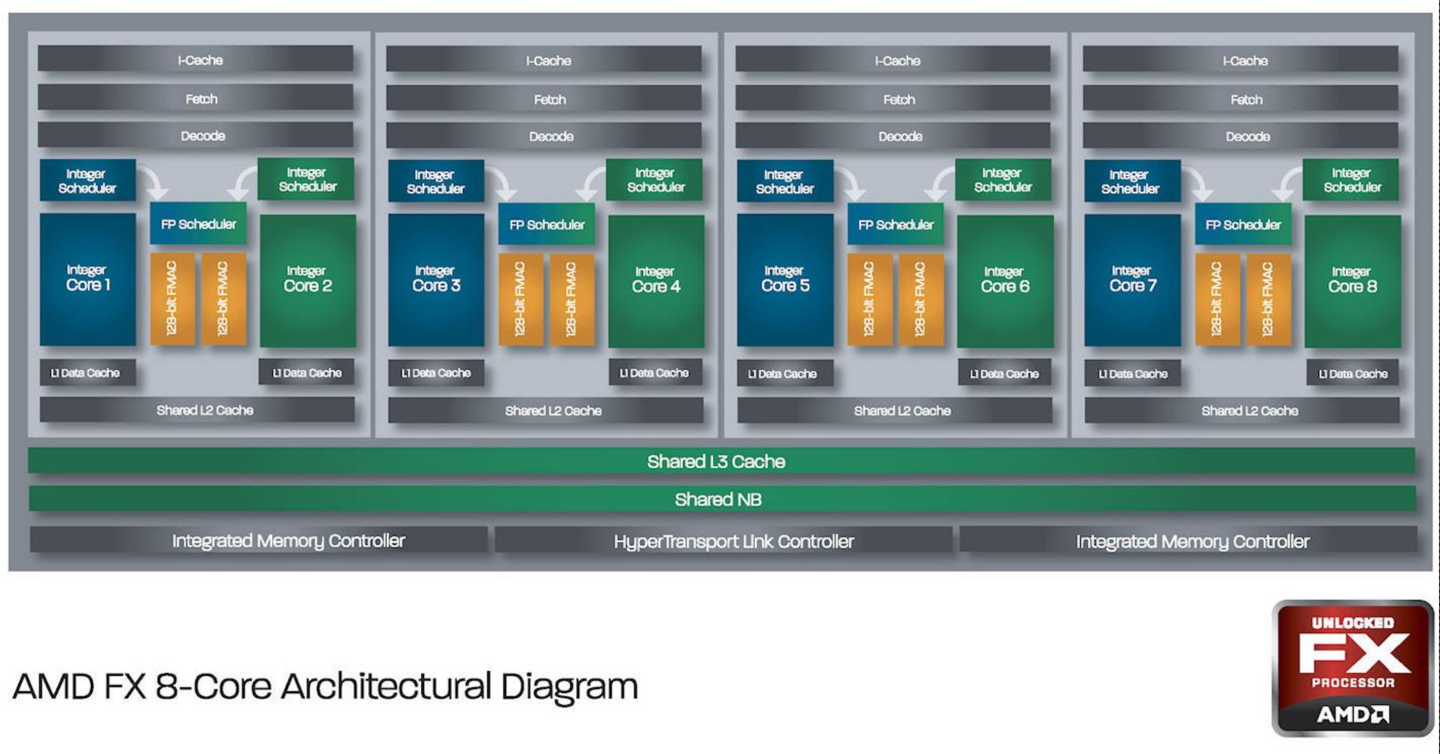
Parallel computing platforms (cont.)



A generalized compute node

- (Possibly) multiple CPUs
- (Possibly) multiple GPUs

Parallel computing platforms (cont.)



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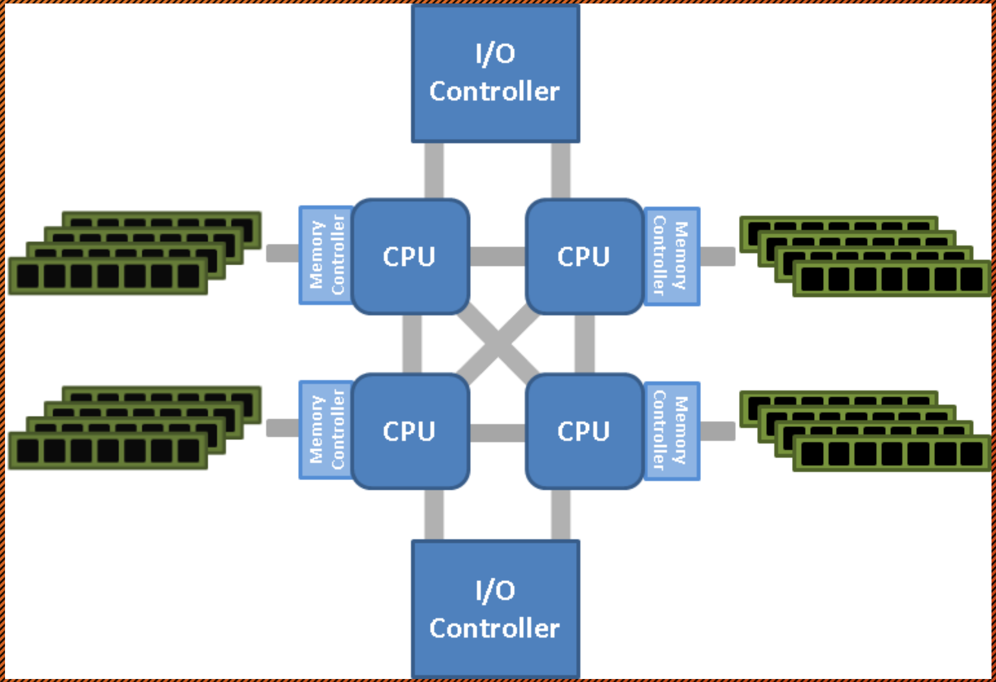
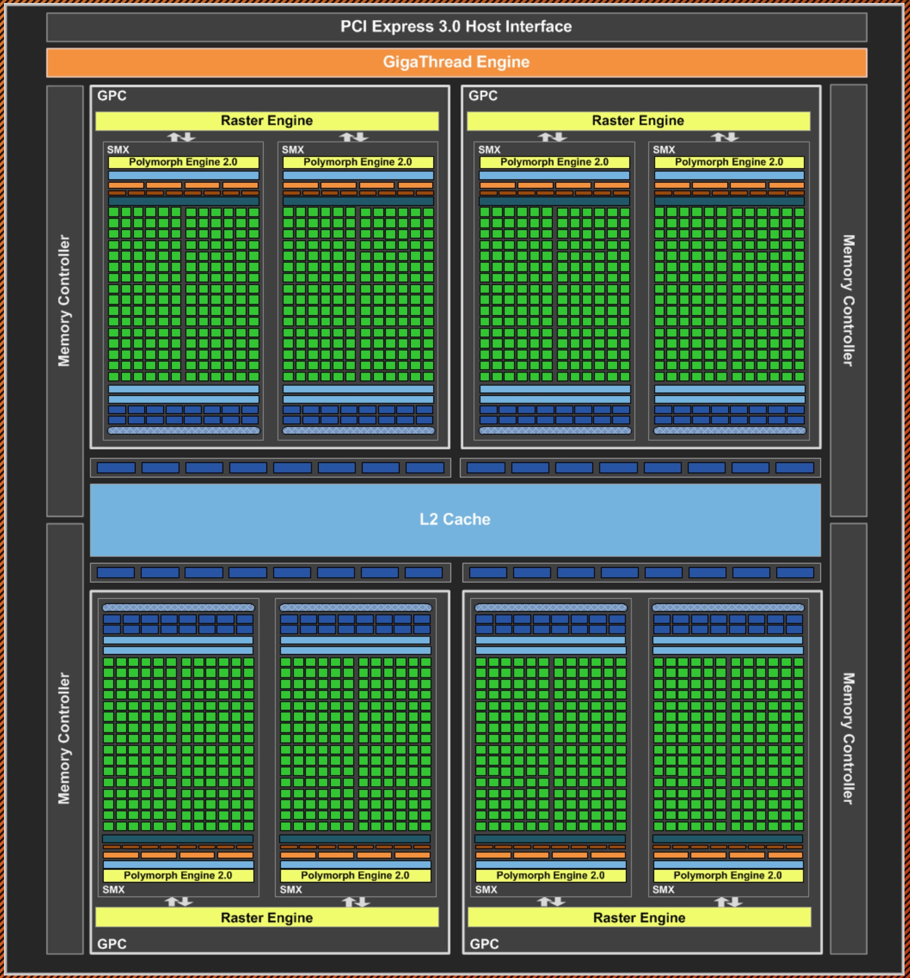


Shared memory on a node, but Non-Uniform Memory Access (NUMA).

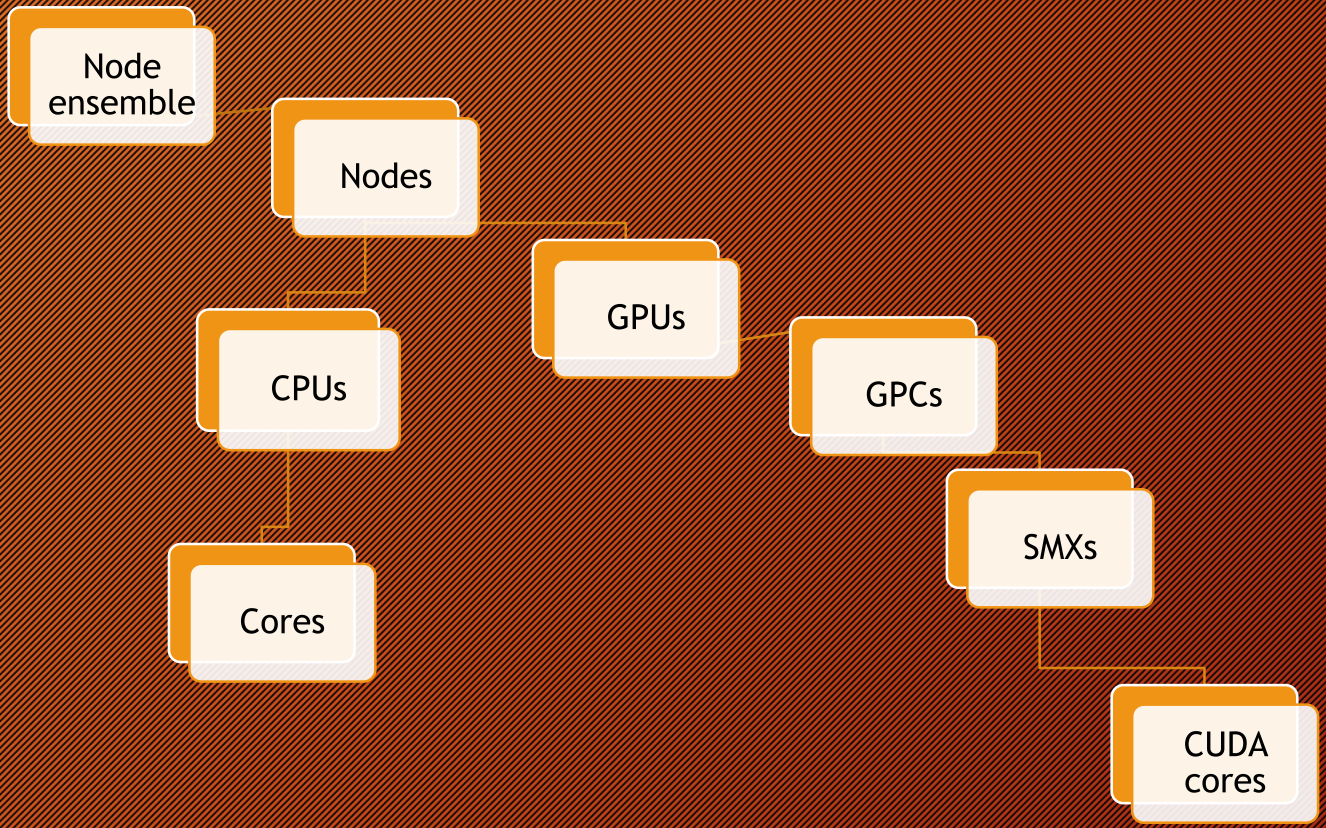
Parallel computing platforms (cont.)



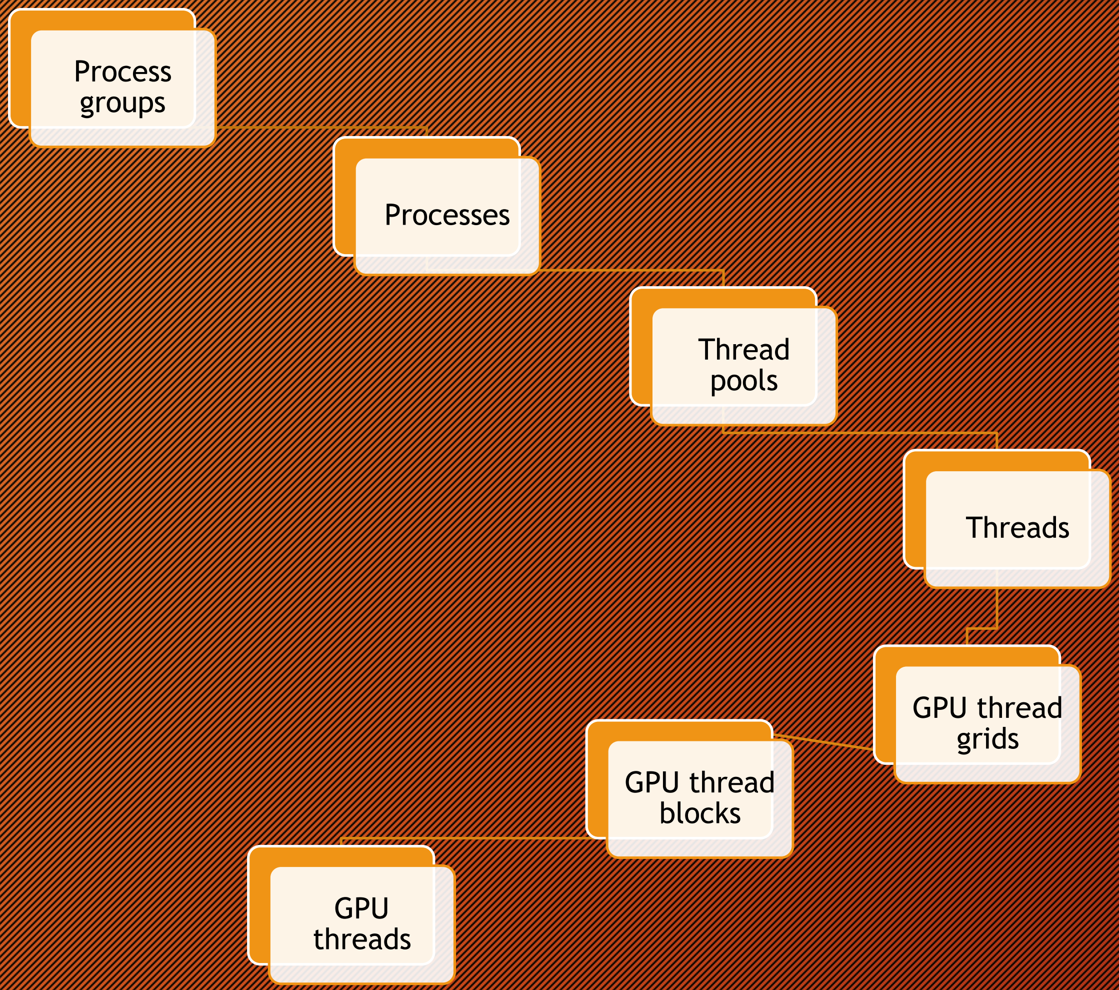
Nvidia GeForce GTX 680 (Kepler)

- 4 GPCs (graphics processing clusters)
- 8 SMXs (streaming multiprocessors)
- 192 X 8 = 1536 CUDA cores

Parallel hardware hierarchy



Parallel program hierarchy



Parallel programming paradigms

<u>Distributed Memory</u> <u>Shared Memory</u> <u>Accelerator</u>

Parallel programming paradigms

<u>Distributed Memory</u>

- Multiple processes
- Distributed address space
- Explicit data movement
- Locality!

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Shared Memory

- Multiple threads
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Shared Memory

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Accelerator

- Host memory ← PCIe → Device memory
- Explicit data movement
- Locality!

Algorithm design

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Parallel algorithm design steps

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- Synchronizing computing agents for correct program execution

Parallel algorithm design steps

- Identifying portions of work that can be performed concurrently - Decomposition
- Mapping concurrent pieces of work onto computing agents running in parallel
- Making the input, output, and intermediate data available to the right computing agent at the right time – Data Dependencies
- Managing simultaneous requests for shared data
- Synchronizing computing agents for correct program execution – Task Dependencies

Decomposition for concurrency

Task Decomposition

Data Decomposition

Decomposition for concurrency

Task Decomposition

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Data Decomposition

- Data is partitioned (input, output, or intermediate)

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Data Decomposition

- Data is partitioned
- Partitions are assigned to computing agents
- “Owner computes” rule

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Data Decomposition

- Data is partitioned
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- Usually static

Decomposition for concurrency

Data Decomposition + Task Decomposition = Hybrid Decompositions

(Example: sparse matrix factorization)

Task decomposition example

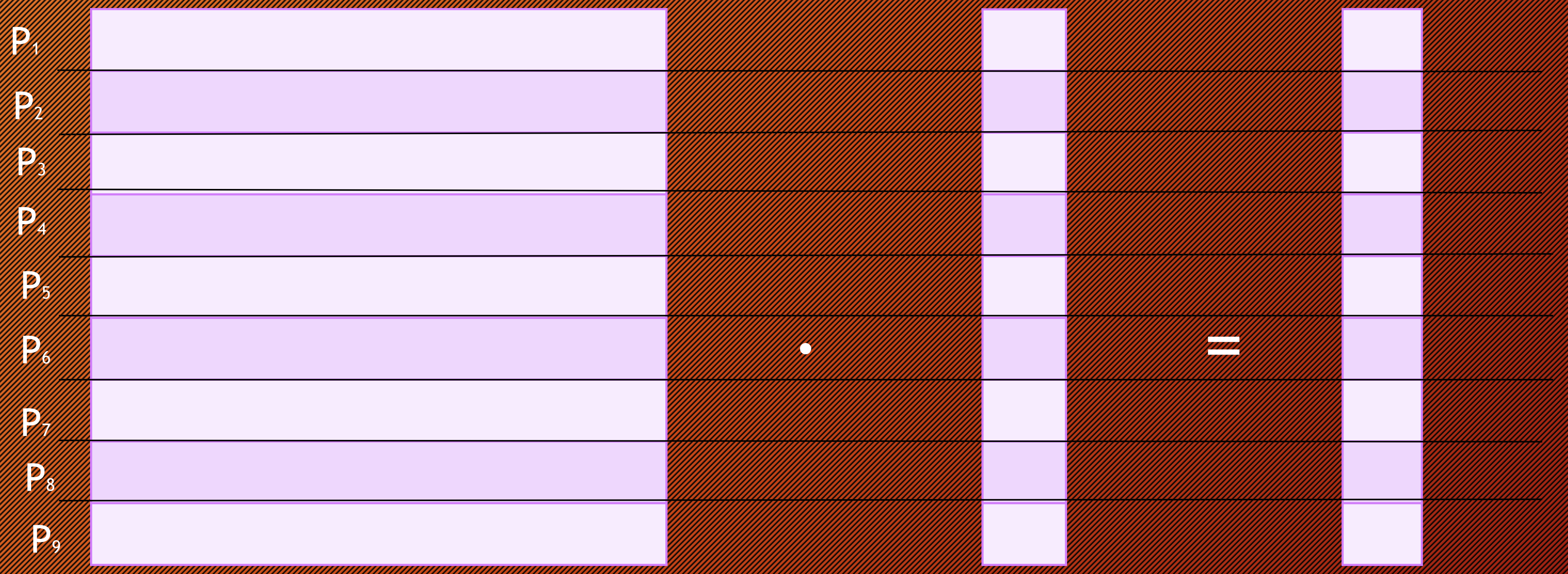
Chess Program

R1 K1 B1 Q . . . P1 P2

- Each task evaluates all moves of a single piece (branch-and-bound)
- Small data (board position) can be replicated
- Dynamic load balancing required

Data decomposition example

Dense Matrix-Vector Multiplication



Parallel application design guidelines

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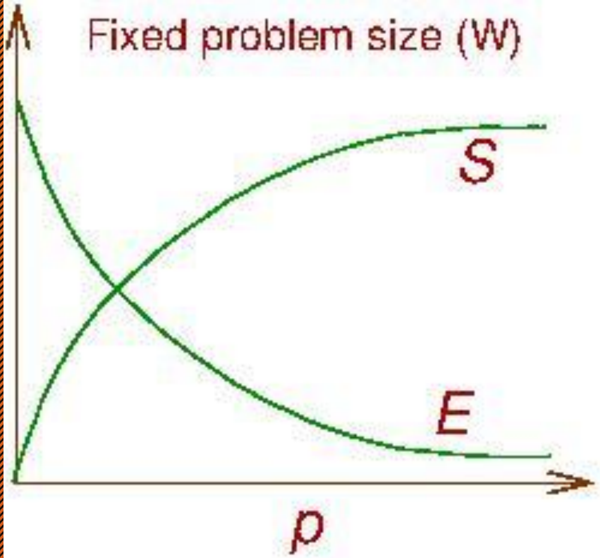


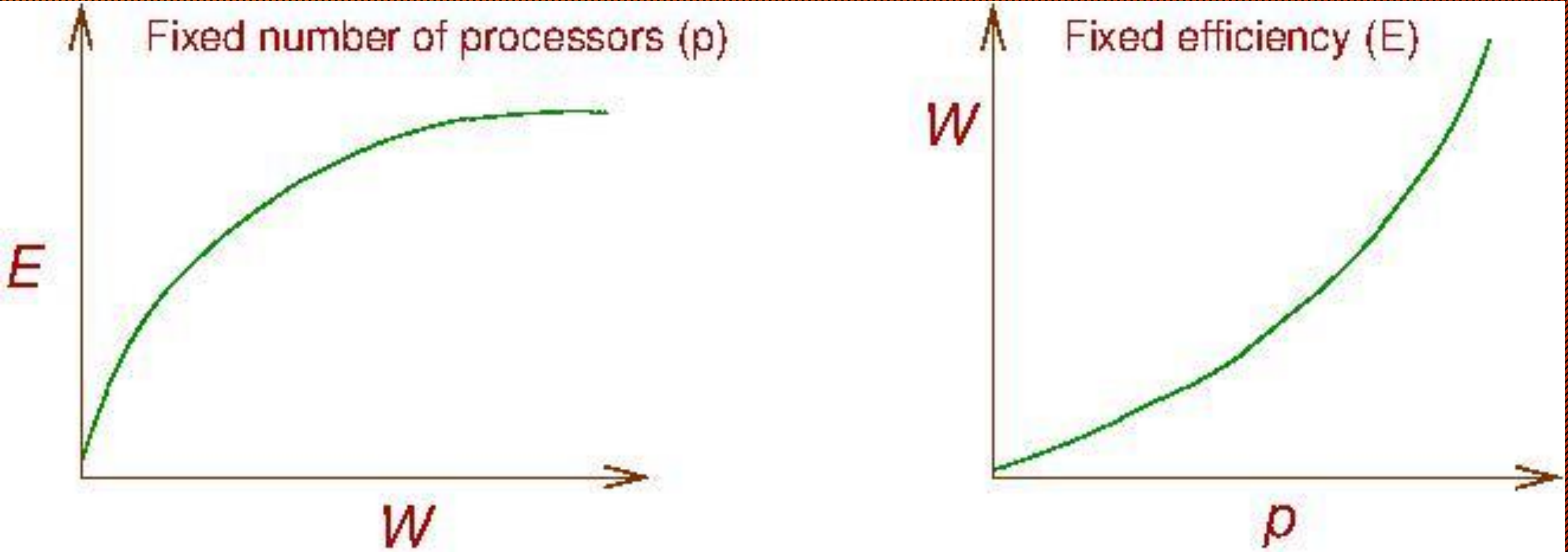
W = problem size (opcount)

p = number of computing agents

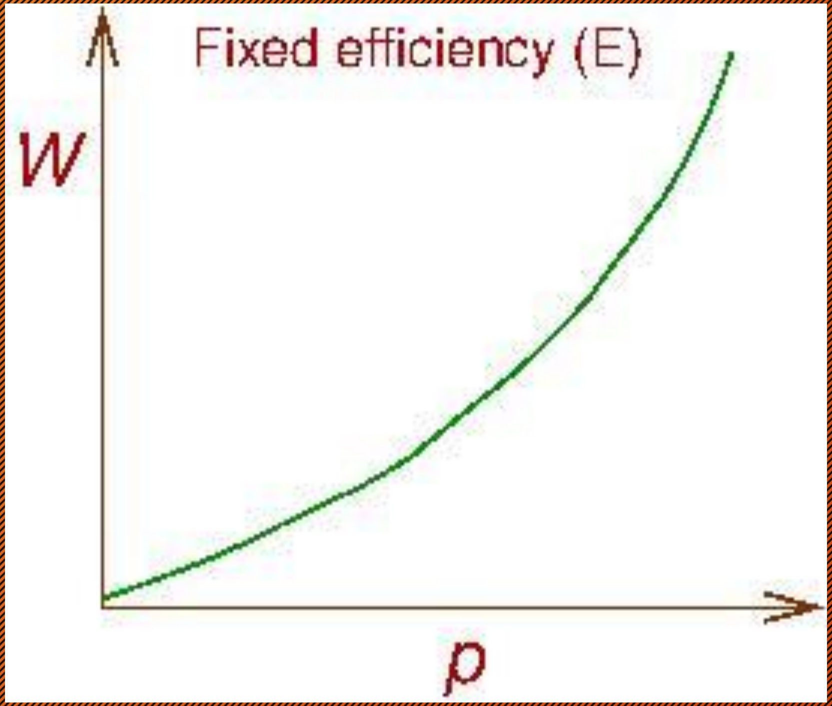
S = speedup

E = efficiency = S/p

Parallel algorithm analysis

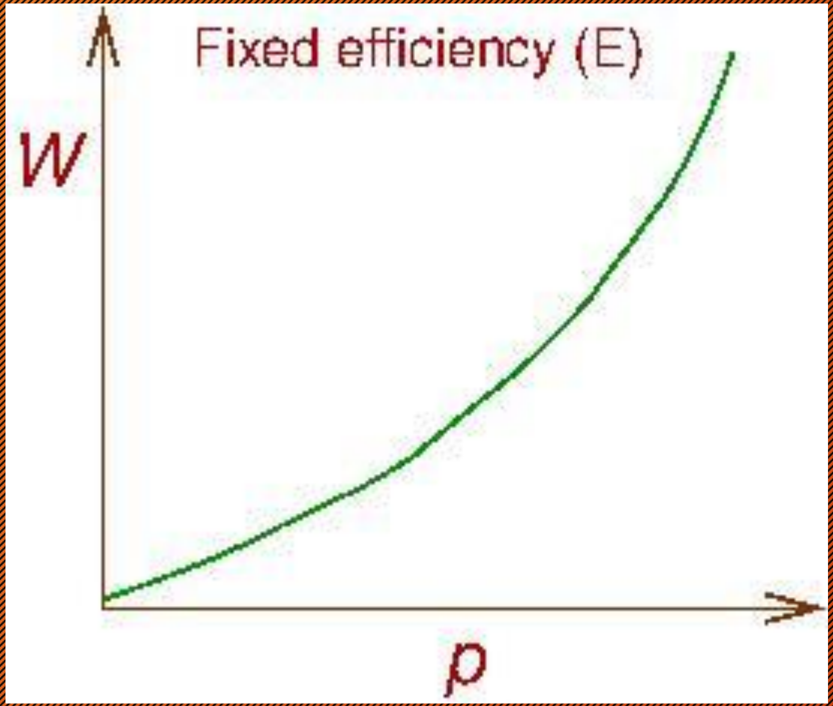


Isoefficiency function



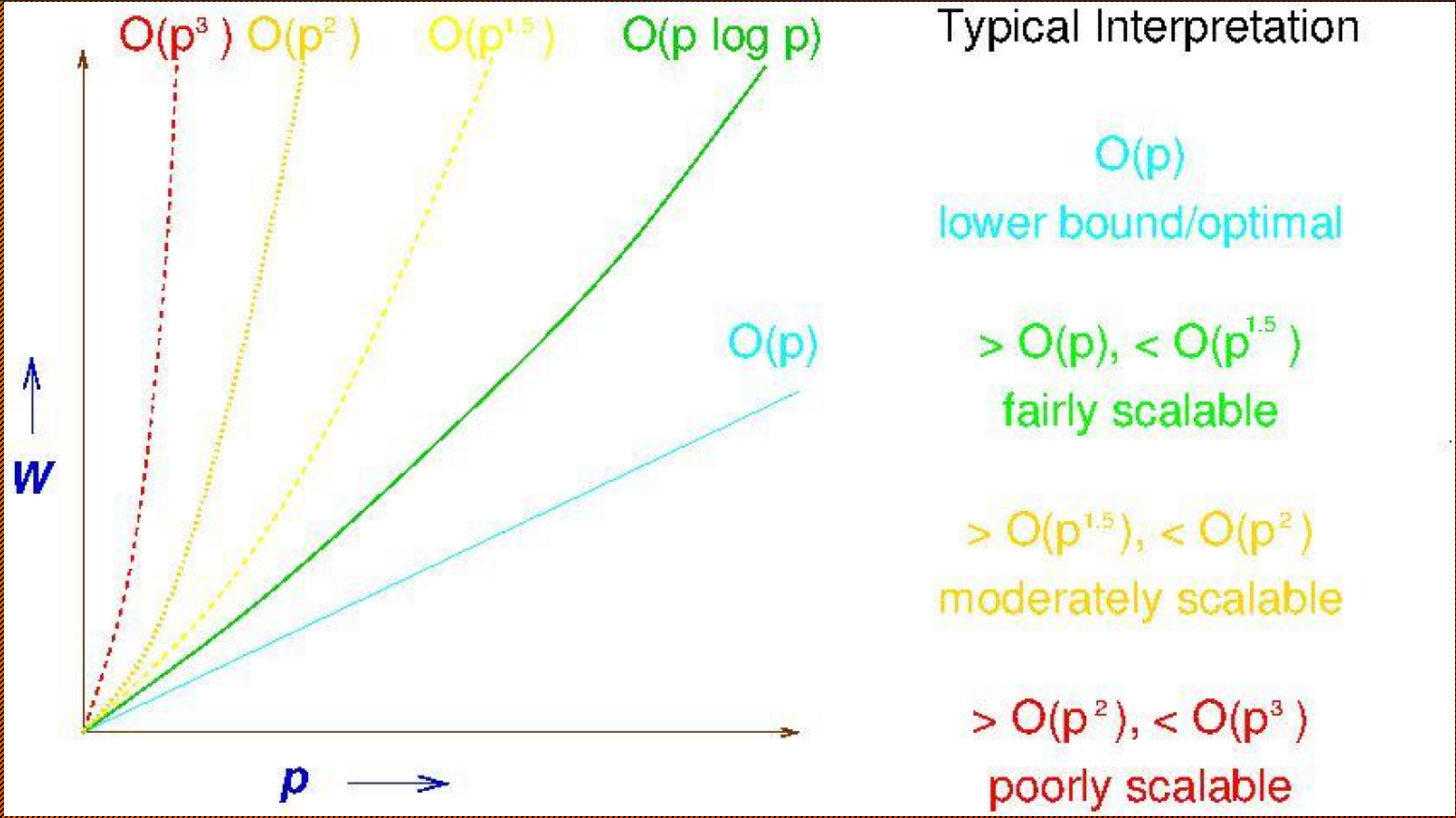
- Function $f_E(p)$ of the number of computing agents p by which the problem size W must grow in order to maintain a given efficiency E.

Isoefficiency function



- Function $f_E(p)$ of the number of computing agents p by which the problem size W must grow with p in order to maintain a given efficiency E.
- Captures the effect of communication, load-imbalance, contention, serial-bottlenecks, etc.

Isoefficiency function



Scalability analysis

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$W = T_O.E/k(1-E)$

$W \sim T_O$

Scalability analysis: $W = O(n^3)$

Algorithm A

$T_P = O(n^3/p) + O(n^2/\sqrt{p})$

Algorithm B

$T_P = O(n^3/p) + O(n\sqrt{n})$

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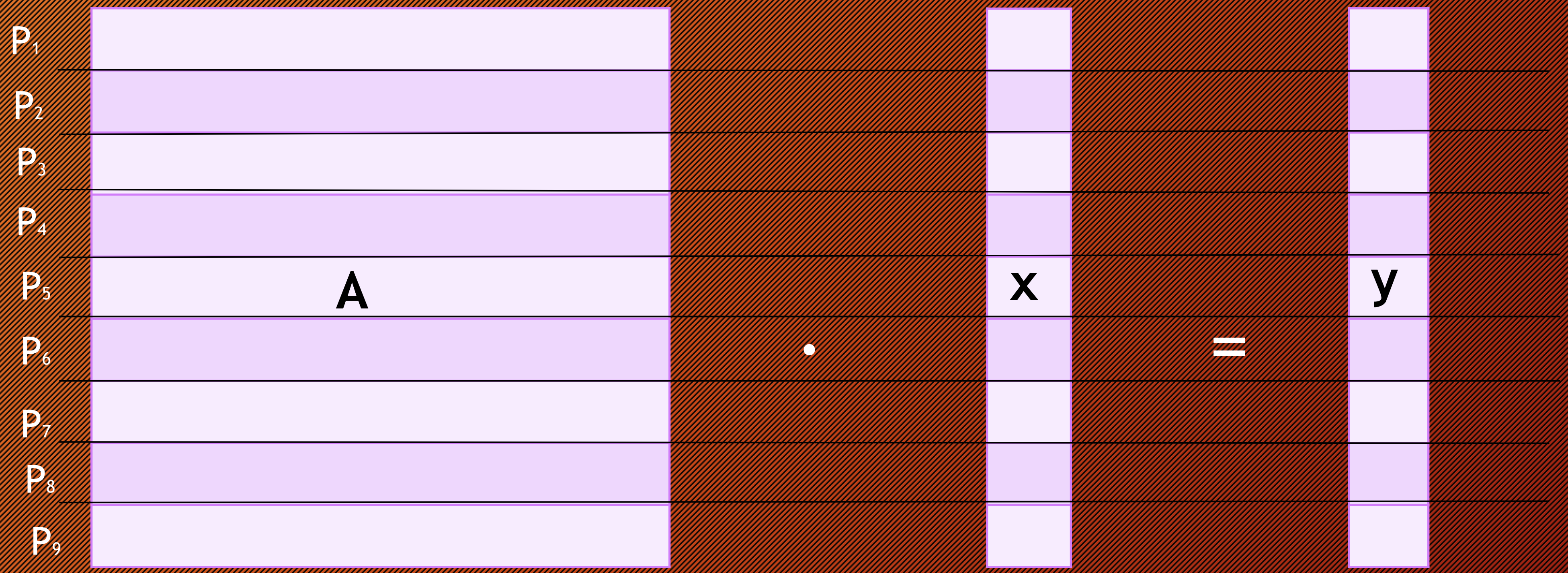
$n^{1.5} \sim p$

$W = O(n^3) = O(p^2)$

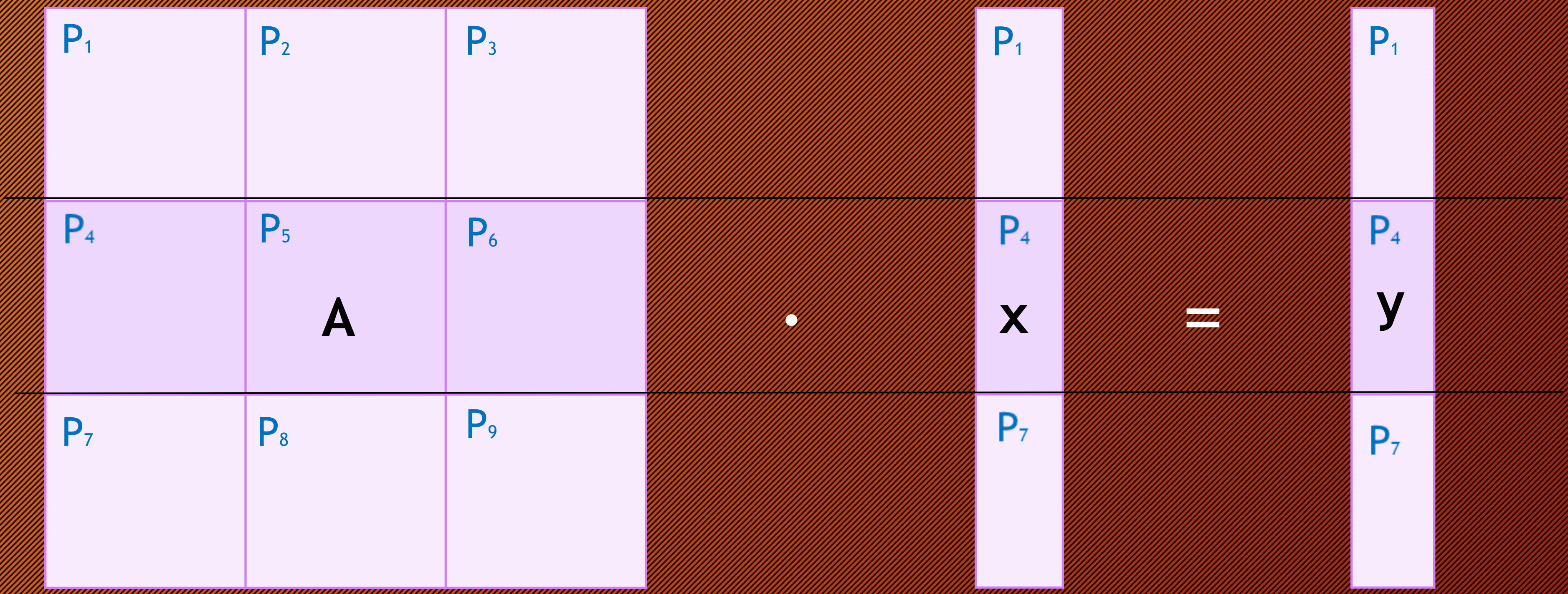
Parallel algorithm design and analysis

Dense Matrix-Vector Multiplication (1-D decomposition)

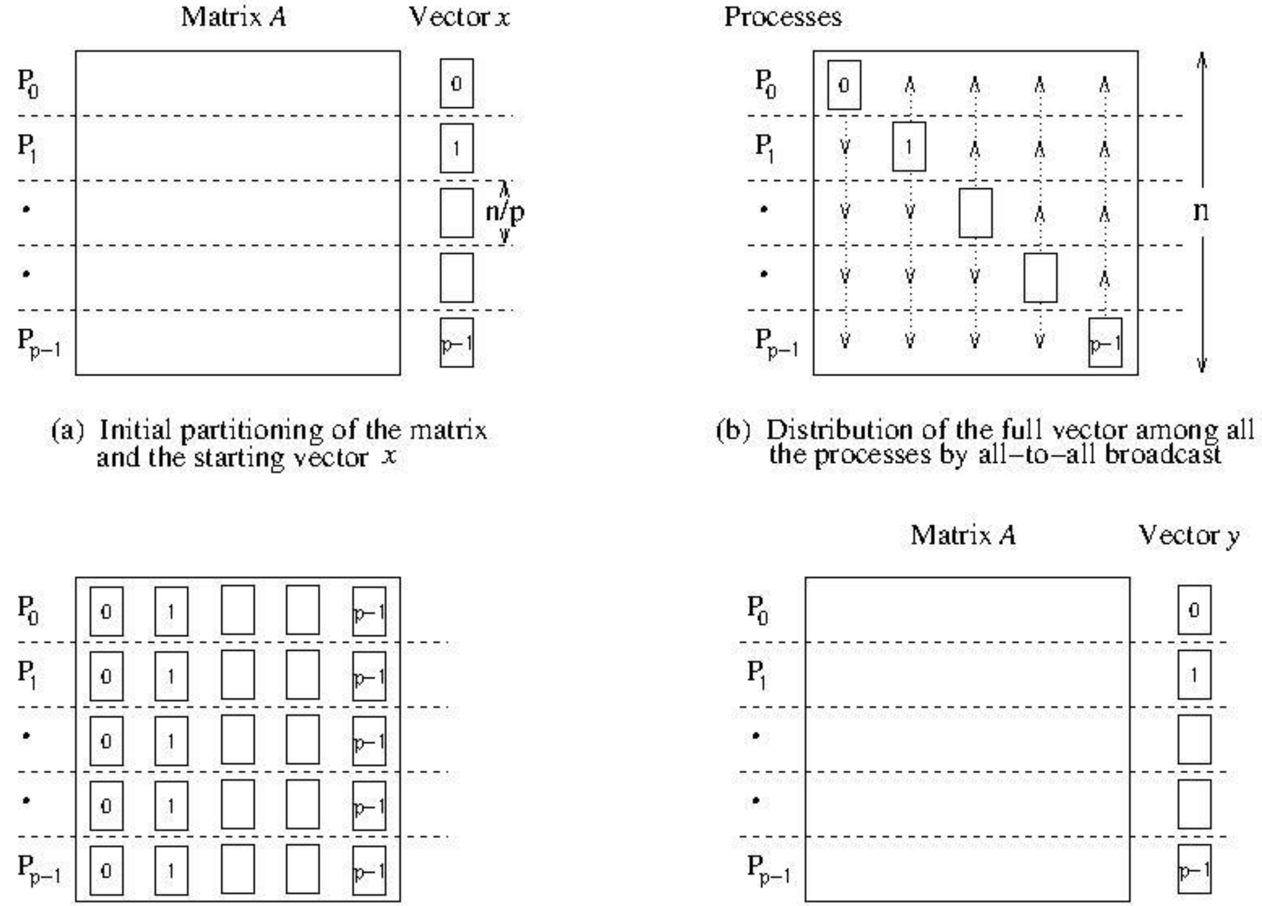


Parallel algorithm design and analysis

Dense Matrix-Vector Multiplication (2-D decomposition)



Parallel matrix-vector multiplication: 1-D decomposition



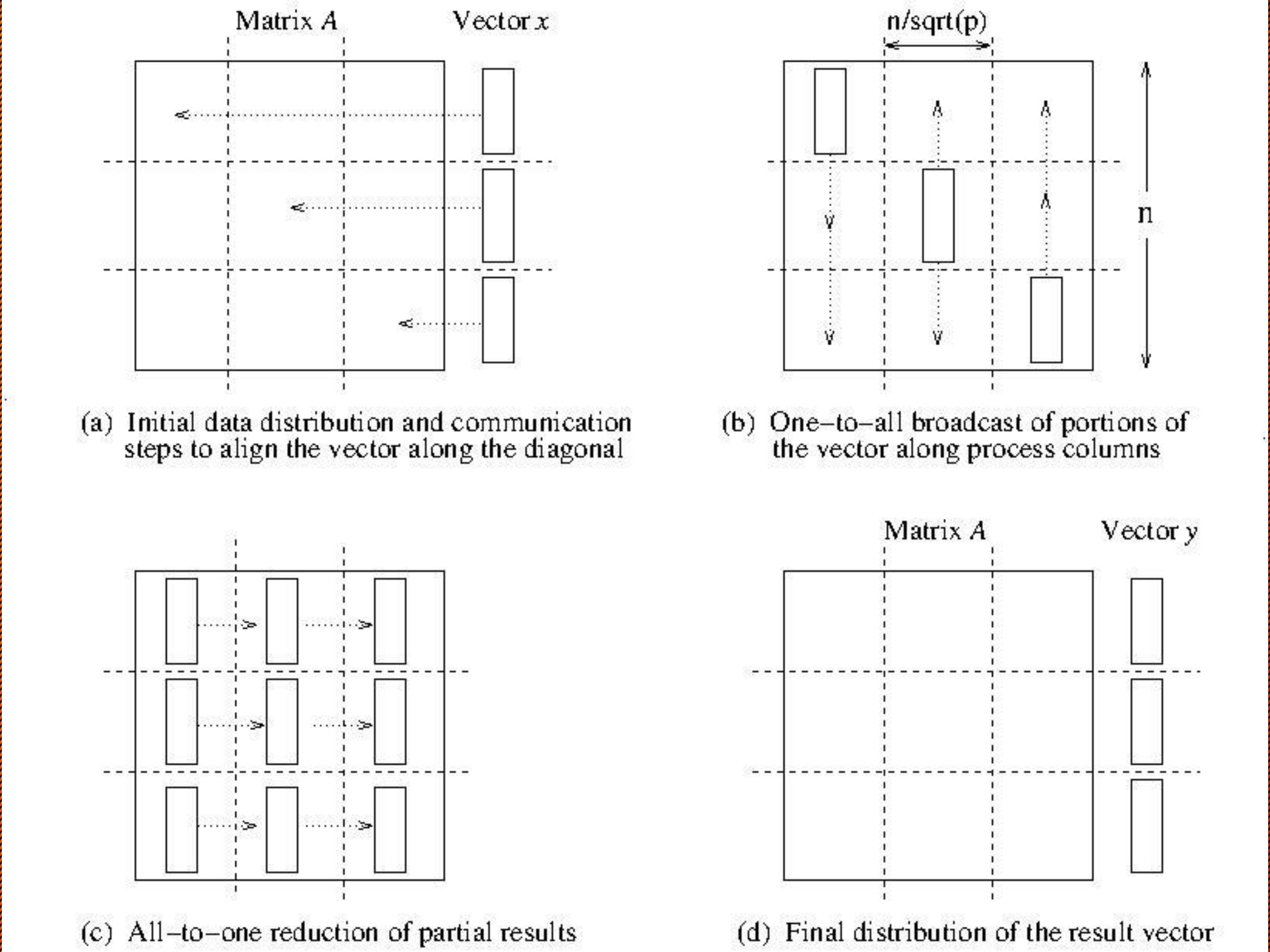
(a) Initial partitioning of the matrix and the starting vector x

(b) Distribution of the full vector among all the processes by all−to−all broadcast

(c) Entire vector distributed to each process after the broadcast

(d) Final distribution of the matrix and the result vector y

Parallel matrix-vector multiplication: 2-D decomposition



Scalability analysis of matrix-vector multiplication (1-D decomposition)

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Isoefficiency function of dense sparse matrix-vector multiplication

1-D decomposition

$W \sim p^2$

2-D decomposition

$W \sim p\log^2 p$

2-D decomposition is likely to yield higher speedups, require smaller problems to deliver the speedups, and scale more readily to larger number of computing agents.

Concluding remarks

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- Analysis important to understand scalability

Thank you!