# Stability of causal inference

Leonard J. Schulman & Piyush Srivastava

California Institute of Technology

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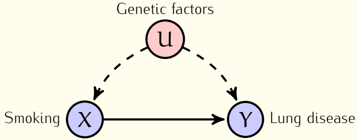
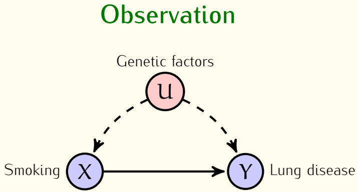
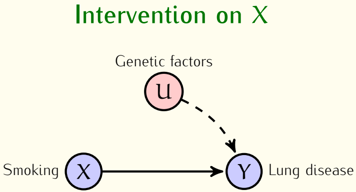
# Causal identification: Experimental intervention

## Observation



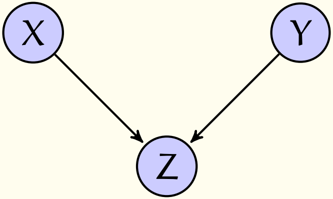
# Causal identification: Experimental intervention



# Directed graphical models

- A directed acyclic graph $G = (V, E)$ whose nodes are random variables
- Absent edges represent conditional independence assumptions

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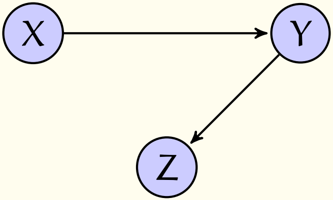


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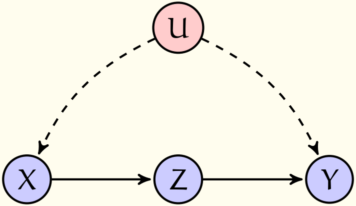
# Semi-Markovian models

- A Markovian model with some nodes hidden
- Hidden nodes have no parents




## Observed distribution

$$P(X, Y, Z) := \sum_{u} P(U = u)P(X|U = u)P(Z|X)P(Y|X, U = u)$$

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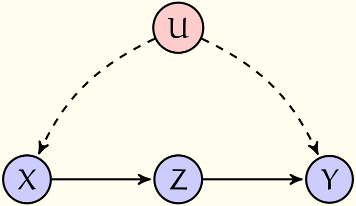


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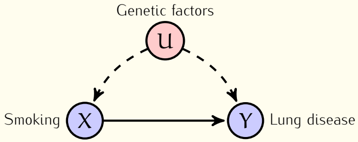
# Interventions without experiments [Pearl, 1995]

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$$P(X, Y)$$

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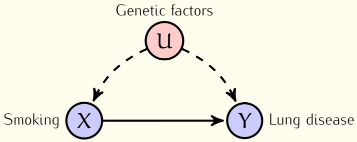


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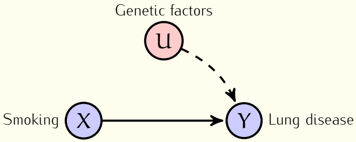


### Intervention distribution

$$P(Y \mid do(X = x))$$

$$\sum_{u} P(U = u) P(Y|X = x, u)$$

### Identification problem [Pearl, 1995]

When is $P(Y = y | do(X = x))$ computable given the observed distribution $P$?

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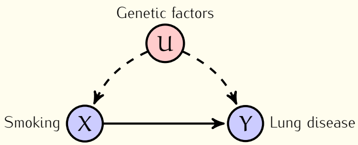


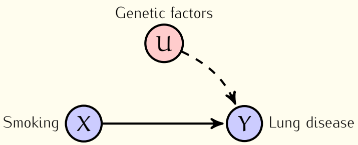


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[**Pearl, 1995**]

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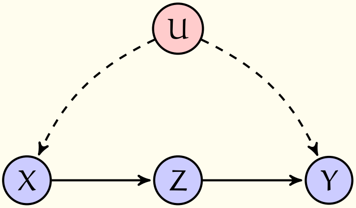
**Not always!**

# Identifiable models

## But sometimes it is...




### Identification

$$P(Y \mid do(X = x)) = \sum_z P(Z = z | X = x) \cdot \sum_{x'} P(X = x') P(Y = y \mid Z = z, X = x').$$

# Deciding identifiability

A long line of work culminated in the following striking result

**Complete Identification** [Huang and Valtorta, 2008; Shpitser and Pearl, 2006, ...]

An efficient algorithm with the following characteristics exists:

Input: Semi-Markovian graph $G = (V, E, U, D)$, disjoint subsets $X, Y$ of $V$

Output: Either

- A rational map

$$\mathsf{ID}(G, X, Y) : P(V) \mapsto P(Y \mid \mathsf{do}(X)), \text{ or}$$

- A certificate of non-existence of such a map

**Note**

- The observed distribution $P$ is not an input to the algorithm
- The output is not numerical, but a symbolic, exact description of the map

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**ID assumes...**

- Exact knowledge of observed distribution $P$
- Exact knowledge of the model $G$ (no "missing" edges)

# Stability of the identification map

$$G = (V, E, U, D) \text{ is a semi-Markovian graph}$$
$$\mathsf{ID}(G, X, Y) : P(V) \mapsto P(Y \mid \mathsf{do}(X))$$

**Statistical stability**

How sensitive is $\mathsf{ID}(G, X, Y)$ to small perturbations in the input $P$?

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## Statistical stability

How sensitive is $\mathsf{ID}(G, X, Y)$ to small perturbations in the input $P$?

## Model Stability

How sensitive is $\mathsf{ID}(G, X, Y)$ to extra assumptions (missing edges) in $G$?

# Perturbations in the input: Condition number

$$G = (V, E, U, D) \text{ is a semi-Markovian graph}$$
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Suppose instead of $P$, we get $\tilde{P}$ as input to $\mathsf{ID}(G, X, Y)$, such that

$$(1-\epsilon) \leqslant \frac{\tilde{P}(\cdot)}{P(\cdot)} \leqslant (1+\epsilon) \quad \equiv \quad \mathsf{Rel}\,P \leqslant \epsilon, \text{ in } \|\cdot\|_\infty \text{ norm}$$

## Condition number

$$\kappa_{\mathsf{ID}(G,X,Y)} = \qquad \sup \quad \frac{\mathsf{Rel}\,P(Y|\,\mathsf{do}(X))}{\mathsf{Rel}\,P}$$

How large is the relative error in the output compared to that in the input?

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e.g., $\kappa$ for computing conditional probabilities from $P$ is at most $2$.

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## Sources of perturbations

- Standard model for floating-point round off in numerical analysis
- Statistical sampling errors: usually additive (even worse)
- Intentionally introduced errors: e.g. by some differential privacy mechanisms

# Perturbations in the input: Inaccurate models

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## Ignoring "weak" edges

The same framework of perturbations to $P$ can handle "model stability" as well!

[see paper for details]

# Results: Condition of causal identification

**Theorem: There exist highly ill-conditioned examples!**

There exists an infinite sequence of semi-Markovian graphs $G_n$ with $n$ observed vertices and disjoint subsets $S_n$ and $T_n$ of the observed vertices such that

$$\kappa_{\mathsf{ID}(G_n, T_n, S_n)} = \exp\left(\Omega\left(n^{0.49}\right)\right)$$

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On these examples, any algorithm computing **ID** may lose

$\Omega\left(n^{0.49}\right)$

bits of precision

Condition vs. Stability

# Results: Condition of causal identification

**Theorem: An important class of well-conditioned examples**

Let $G$ be a semi-Markovian graph and let $X$ be an observed node in $G$ such that it is not possible to reach a child of $X$ from $X$ using only the hidden edges. Then, for any subset $S$ of $V$ not containing $X$.

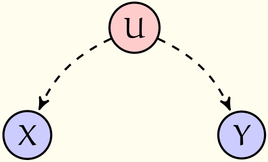
$$\kappa_{\mathsf{ID}(G,X,S)} = O(|V|).$$

- Identifiability under the above condition was proved by Tian and Pearl [2002]

Ill-conditioned examples

# Primitives of identifiability

# Easy cases: no directed edges

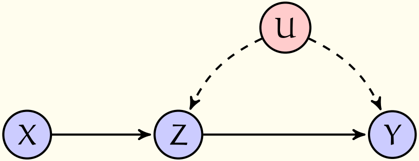



## Identification

$$P(Y \mid \mathrm{do}(X = x)) = \sum_x P(Y, X = x) = P(Y)$$

In general, if $X$ is not an ancestor of $Y$, it can be marginalized

# Easy cases: no hidden edges

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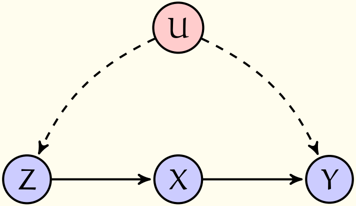


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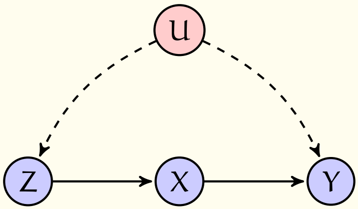


## Identification

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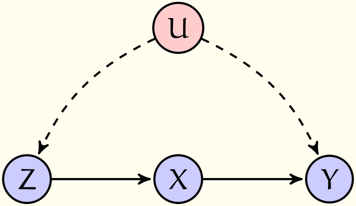
- ...and also, in connivance with the innocuous marginalization described above, the main source of ill-conditioning!

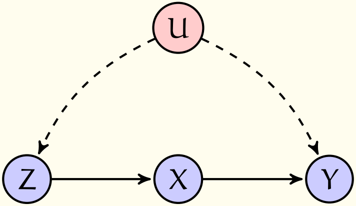
# C-components




## C-components [Tian and Pearl, 2002]

$\{Y, Z\}$ in the above graph is a C-component: a maximal connected component among observed nodes induced by the hidden edges

# C-components

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## C-components are identifiable [Tian and Pearl, 2002]

If $S \subseteq V$ is a C-component in $G = (V, E, U, D)$ then

$$P(S \mid \mathrm{do}(V - S)) = \prod_{A \in S} P(A \mid V_{\pi(<A)}),$$

where $\pi$ is a topological order on $V$ according to $E$

# C-components and general identifiability

## The hardest case

The “hardest” case for identifiability is $P(S|\mathsf{do}(X))$, where

- $X$ is an ancestor set for $S$ in $G$, and
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Case 1 **$S \cup X$ is a C-component in $G$:** $\mathbf{ID}(G, S, X)$ does not exist

Case 2 **$S$ is a C-component in $G$:** Use C-component identifiability

Case 3 **$S \cup X'$ is a C-component in $G$, for some $X' \subsetneq X$:**

## Recursion

Call $\mathbf{ID}(S \cup X', X', S)$, but with $P$ replaced by

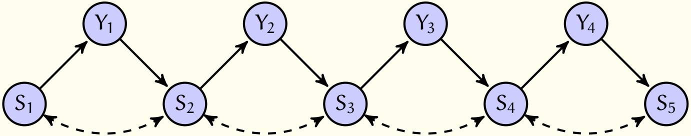
$$P'(S \cup X') := \prod_{A \in S \cup X'} P(A \mid V_{\pi(<A)}),$$

where $\pi$ is a topological order on $V$ according to $E$

Recursion will fail immediately unless some $X'$ is no more an ancestor of $S$!

# The ill-conditioned examples

# A warm-up calculation: $\kappa$ is at least $\Omega(n)$

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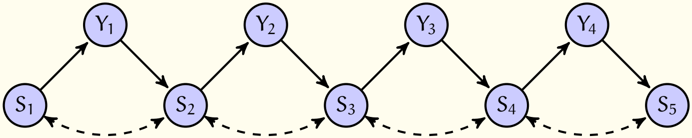
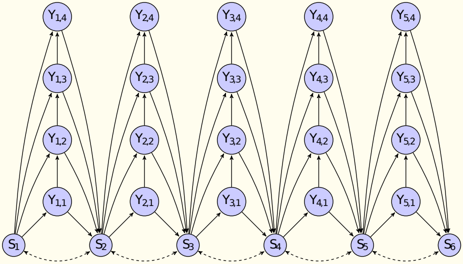



$$P(\cdot) \mapsto P(\boldsymbol{S} \mid \mathrm{do}(\boldsymbol{Y})) = P(.) \mapsto \prod_{i=1}^{n} P(S_i | S_{<i} Y_{<i})$$

- When $P$ is uniform, the output of the map is the uniform distribution
- However, one can construct a $\tilde{P}$ that is $\epsilon$-close to $P$ and such that each conditional probability above has a positive $\Omega(\epsilon)$ relative error,
  - for a total relative error of $\Omega(n\epsilon)$.

No recursion was used here!

# The final gadget $(m = 6, k = 4)$: $P(\boldsymbol{S} \mid do(\boldsymbol{X}, \boldsymbol{Y}))$



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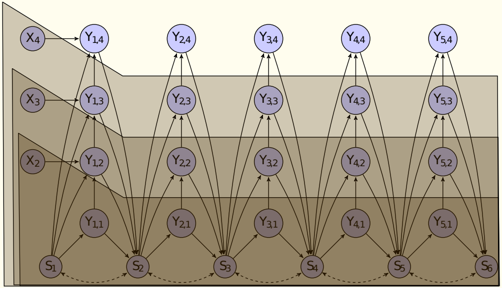


$$P\left(X_{[k]}, S_{[m+1]}, Y_{[m],[k]}\right) := \quad P\left(X_k = x, X_{[k-1]}\right) \cdot \prod_{i=1}^{m} P\left(S_i, Y_{i,[k]} \mid \mathrm{pred}_i\right) \cdot P\left(S_{m+1} \mid \mathrm{pred}_{m+1}\right),$$

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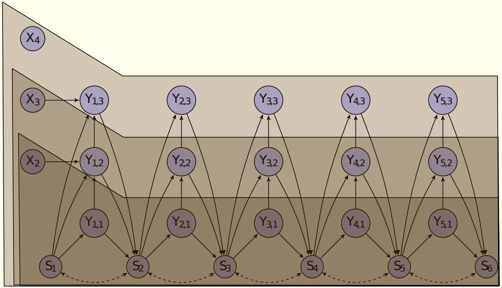


$$P'\left(X_{[k]}, S_{[m+1]}, Y_{[m],[k-1]}\right) := \quad P\left(X_k = x, X_{[k-1]}\right) \cdot \prod_{i=1}^{m} P\left(S_i, Y_{i,[k-1]} \mid \text{pred}_i\right) \cdot P\left(S_{m+1} \mid \text{pred}_{m+1}\right),$$

$\text{Rel}\, P = \epsilon \quad \rightsquigarrow \quad \text{Rel}\, P' \sim m \cdot \epsilon$

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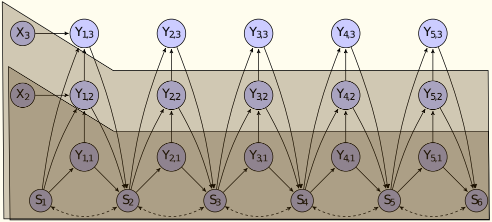


$$\pi(P)\left(X_{[k-1]}, S_{[m+1]}, Y_{[m],[k-1]}\right) := \sum_x P\left(X_k = x, X_{[k-1]}\right) \cdot \prod_{i=1}^{m} P\left(S_i, Y_{i,[k-1]} \mid \mathrm{pred}_i\right)$$
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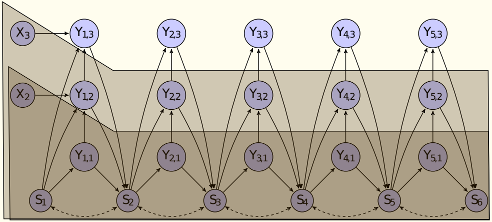


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Repeat $k$ times to get **Rel ID** $\sim m^k \cdot \epsilon$?

# Comments

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- To get a condition number of $\sim \Omega(\exp(\sqrt{n}))$, choose $m \approxeq k \approxeq \sqrt{n}$

Details of analyzing this correctly are somewhat involved: please see paper

# Conclusion

**Condition number of causality**

**Highly ill-conditioned examples exist**

Very small uncertainties in the model or data can introduce very large errors in causal identification

**But not all instances are ill-conditioned**

A well studied class of examples indeed has small condition number: so numerically stable algorithms can be designed

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# Condition number and numerical stability

Condition number is a property of the function
Numerical stability is a property of a floating point algorithm

$$\mathsf{ADD} : (x_1, x_2, \ldots, x_n) \mapsto x_1 + x_2 \ldots x_n$$

Condition number

$$\kappa = \frac{\sum_{i=1}^{n} |x_i|}{|\sum_{i=1}^{n} x_i|} = 1, \text{ for positive } x_i$$

Numerical stability: Naive linear summation

$$O(n \cdot \varepsilon \cdot \kappa)$$

Numerical stability: Kahan summation

$$O(\varepsilon \cdot \kappa), \text{ to first order in } \varepsilon$$

**$\varepsilon$ is the "machine epsilon"**

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