A APPENDIX

Proof sketch of Corollary 4.5. The proof follows similarly to Theorem 4.2. But, here the terms $f_t^i(y, \theta)$ and $f_t^o(y, \theta)$ are slightly different

$$\begin{aligned} f_t^i(y,\theta) &= c^d d(0,y) + \lambda \mathbb{E}\left[\max\{-c^d d(0,y) + L(y), c^d d(y,-y) + L(y)\}\right] \\ &= \mathbb{E}\left[\max\{c^d(1-\lambda)d(0,y) + L(y), c^d\left(d(0,y) + \lambda d(y,-y)\right) + L(y)\}\right] \end{aligned}$$

By properties A1, A2, and modified A3 and $\lambda < 1$, all terms in the right hand side are convex. The maximization and expectation preserve convexity. Thus, $f_t^i(y, \theta)$ is convex. The convexity of $f_t^o(y, \theta)$ can be shown in the same way using the following recurrence

$$\begin{aligned} f_t^o(y,\theta) &= -c^d d(y,-y) + \lambda \mathbb{E}\left[\max\{-c^d d(0,y) + L(y), c^d d(y,-y) + L(y)\}\right] \\ &= \mathbb{E}\left[\max\{-c^d(d(y,-y) + \lambda d(0,y)) + L(y), c^d(\lambda - 1)d(y,-y) + L(y)\}\right] \end{aligned}$$

Now, we show that when t goes to infinity, the function $f_t^i(y, \theta)$ is still convex. The basic idea is that it has been shown in Putman's book that the operation that maps $f_t^i(y, \theta)$ to $f_{t-1}^i(y, \theta)$ is a contraction under maximum norm. Therefore, by Banach fixed point theorem, there exists a fixed point denoted by $f_\infty^i(y, \theta)$. And, since the contraction is under maximum norm, we have that the function $f_t^i(y, \theta)$ uniformly converges to $f_\infty^i(y, \theta)$. Since each $f_t^i(y, \theta)$ is convex, $f_\infty^i(y, \theta)$ is also convex under uniformly convergence. It is similar that when t goes to infinity, the function $f_t^o(y, \theta)$ is still convex. Therefore, we have two threshold policy as the optimal stationary policy. $\square$

A.1 Transition Probabilities

In order to make the results reproducible, we include the precise definition of the transition matrix P that describes the price process:

0.815584	0.176623	0.005195	0.002597	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.047011	0.872398	0.072532	0.007388	0.000000	0.000672	0.000000	0.000000	0.000000	0.000000
0.001199	0.131894	0.779376	0.069544	0.014388	0.003597	0.000000	0.000000	0.000000	0.000000
0.000000	0.042453	0.283019	0.514151	0.122642	0.023585	0.009434	0.000000	0.000000	0.004717
0.000000	0.021505	0.096774	0.268817	0.430108	0.129032	0.043011	0.010753	0.000000	0.000000
0.000000	0.000000	0.032258	0.258065	0.354839	0.193548	0.096774	0.064516	0.000000	0.000000
0.000000	0.071429	0.142857	0.000000	0.071429	0.214286	0.285714	0.142857	0.071429	0.000000
0.000000	0.000000	0.142857	0.000000	0.285714	0.000000	0.000000	0.285714	0.285714	0.000000
0.000000	0.000000	0.000000	0.000000	0.250000	0.250000	0.250000	0.000000	0.250000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000

The rows represent the start states and the columns represent the end states of the transitions. The first state represents the price of \$ 25 and the price for each consecutive state increases by \$ 25.