
Nested Markov Properties for Acyclic Directed Mixed Graphs

Thomas S. Richardson
thomasr@u.washington.edu

James M. Robins
robins@hsph.harvard.edu

Ilya Shpitser
ishpitse@hsph.harvard.edu

Abstract

Directed acyclic graph (DAG) models may be characterized in four different ways: via a factorization, the d-separation criterion, the moralization criterion, and the local Markov property. As pointed out by Robins [2, 1], Verma and Pearl [6], and Tian and Pearl [5], marginals of DAG models also imply equality constraints that are not conditional independences. The well-known ‘Verma constraint’ is an example. Constraints of this type were used for testing edges [3], and an efficient variable elimination scheme [4]. Using acyclic directed mixed graphs (ADMGs) we provide a graphical characterization of the constraints given in [5] via a nested Markov property that uses a ‘fixing’ transformation on graphs. We give four characterizations of our nested model that are analogous to those given for DAGs. We show that marginal distributions of DAG models obey this property.

- [3] Ilya Shpitser, Thomas S. Richardson, and James M. Robins. Testing edges by truncations. In *International Joint Conference on Artificial Intelligence*, volume 21, pages 1957–1963, 2009.
- [4] Ilya Shpitser, Thomas S. Richardson, and James M. Robins. An efficient algorithm for computing interventional distributions in latent variable causal models. In *27th Conference on Uncertainty in Artificial Intelligence (UAI-11)*. AUAI Press, 2011.
- [5] Jin Tian and Judea Pearl. On the testable implications of causal models with hidden variables. In *Proceedings of UAI-02*, pages 519–527, 2002.
- [6] T. S. Verma and Judea Pearl. Equivalence and synthesis of causal models. Technical Report R-150, Department of Computer Science, University of California, Los Angeles, 1990.

References

- [1] James M. Robins. Testing and estimation of direct effects by reparameterizing directed acyclic graphs with structural nested models. In C. Glymour and G. Cooper, editors, *Computation, Causation, and Discovery*, pages 349–405. MIT Press, Cambridge, MA, 1999.
- [2] J.M. Robins. A new approach to causal inference in mortality studies with sustained exposure periods – application to control of the healthy worker survivor effect. *Mathematical Modeling*, 7:1393–1512, 1986.